

AP Calculus AB

Summer Prep Packet

HHHS MATH DEPARTMENT

AP CALCULUS

SUMMER PREP PACKET

Welcome to AP Calculus! This summer review packet includes prerequisite topics from algebra, trigonometry, and geometry that are necessary for success in calculus. You will be given an opportunity to ask questions on these topics in September.

Instructions:

- * Complete the entire packet neatly on loose-leaf (or unlined paper), showing all work. Clearly number each problem.
- * Check your answers and retry problems until you have mastered the contents of this packet.
- * This will be collected the first day of school for a weighted homework grade.
- * Your mastery of these topics will be assessed in September after you have the opportunity to ask questions and attend extra help.

Contents:

- *26 Practice Questions and Answers*
- *Worked-out Solutions will be posted to Google Classroom*

Google Classroom Join Code for 2024 – 2025: **ot35uue**

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- 1) Copy the table and fill-in the blanks with the correct Inequality or Interval Notation.

Inequality	Interval Notation
$-2 \leq x < 3$	i.
ii.	$(-4, 5]$
$-2 \geq x$	iii.
\mathbb{R} (<i>all real numbers</i>)	iv.

- 2) Write the equation for the line in **point-slope form** for each of the following:

- The slope is -4 and the y-intercept is 5.
- The slope is 5 and the line passes through (-6, 71).
- The line that passes through (4, 16) and (-3, 11).
- The line that passes through (-1, 2) and is perpendicular to the line $2x - 3y + 5 = 0$.
- The line that passes through (2, 3) and the midpoint of the segment from (-1, 4) to (3, 2).

- 3) i) Find the point of intersection for the following analytically (using algebra):

$$3x - y - 7 = 0 \quad \text{and} \quad x + 5y + 3 = 0$$

- ii) Find the points of intersection for the following analytically (using algebra):

$$y = x^2 - 2x \quad \text{and} \quad y = 6 - x$$

- 4) Solve the following equations for the indicated variables:

- $A = P + nrP$, solve for P
- $\frac{2x}{4\pi} + \frac{1-x}{2} = 0$, solve for x
- $2x - 2y \frac{dy}{dx} = y + x \frac{dy}{dx}$, solve for $\frac{dy}{dx}$
- $3y^2 y' + 2yy' = 5y' + 2x$, solve for y'

- 5) Use the given functions to evaluate (and fully simplify) each of the following.

$$f(x) = x^2 + 2x + 3 \quad \text{and} \quad g(x) = -2x + 1$$

- $f(-3)$
- $f(x+5)$
- $g(x+h)$
- $\frac{f(x+h)-f(x)}{h}$
- $g(f(x))$
- $f(g(3))$
- $g^{-1}(x)$

- 6 Evaluate and fully simplify $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ for each of the following (Note: Treat Δx like any other variable when evaluating).

i) $f(x) = 2x^2 - x$

ii) $3x + 2$

- 7 Find the domain of each of the following.

i) $f(x) = \frac{x+1}{x^2-4x-21}$

ii) $g(x) = \sqrt{2x-1}$

iii) $h(x) = \frac{5}{\sqrt{x+3}}$

- 8 Write each absolute value function as a piecewise function.

i) $f(x) = |x|$

ii) $g(x) = |x-4|$

iii) $h(x) = \frac{|x|}{x}$

- 9 Simplify each of the following:

i) $\log_2 16$

ii) $\log_3 \frac{1}{27}$

iii) $\log_{64} 4$

iv) $\ln \frac{1}{e^3}$

- 10 Multiply each of the following by its **conjugate** and fully simplify.

i) $3 - 2i$

ii) $4 + \sqrt{7}$

iii) $\sqrt{x+h} - \sqrt{x}$

- 11

Factor completely:

a. $2x^2 - 7x + 3$

c. $x^6 - 16x^4$

b. $x^4 - 1$

d. $4x^3 - 8x^2 - 25x + 50$

- 12 Solve for x in each of the following equations.

a. $4x^2 + 12x + 3 = 0$

c. $\frac{x+1}{x} - \frac{x}{x+1} = 0$

b. $2x + 1 = \frac{5}{x+2}$

- 13) Write the solution set for each of the following inequalities using **interval/set notation**.

i) $(2x + 2)^3 \leq 0$

ii) $(x + 1)^2(x - 2)^3 > 0$

iii) $-x^2(x - 3) < 0$

iv) $\frac{x-2}{x+4} \geq 0$

- 14) Use the table with selected values for functions $g(x)$ and $f(x)$ to evaluate each of the following.

x	$g(x)$	$f(x)$
-2	5	0
0	1	1
1	-2	5
5	0	-2

i) $\frac{f(5)-f(1)}{5-1}$

ii) $f(g(-2))$

iii) $g(g(0))$

iv) $g^{-1}(-2)$

- 15) Factor and fully simplify each of the following. For which x -value(s) does the function have an asymptote and for which does it have a removable discontinuity (hole).

a. $\frac{x^3-9x}{x^2-7x+12}$

b. $\frac{x^2-2x-8}{x^3+x^2-2x}$

- 16)

A water tank has the shape of a cone. The tank is 10m high and has a radius of 3m at the top. If the water in the tank is 5m deep, what is the area of the surface of the water?

- 17)

Two cars start moving from the same point. One travels south at 100 km/h, the other west at 50 km/h. How far apart are they two hours later?

- 18)

i) The hypotenuse of a right triangle is 4 cm and the leg opposite of acute $\angle\theta$ is 2 cm. What is the measure of $\angle\theta$ in degrees and in radians (without the use of a calculator).

ii) The hypotenuse of a right triangle is 8 cm and the leg adjacent to acute $\angle\theta$ is $4\sqrt{2}$ cm. What is the measure of $\angle\theta$ in degrees and in radians (without the use of a calculator).

- 19) Find the value of each of the following without the use of a calculator. Note: You should be able to evaluate each of these in under 20 seconds.

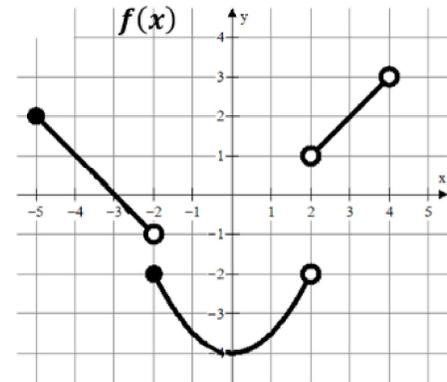
i) $\cos 210^\circ$ ii) $\sin \frac{7\pi}{4}$ iii) $\tan \frac{-2\pi}{3}$ iv) $\cos \frac{5\pi}{6}$ v) $\sin \frac{5\pi}{2}$ vi) $\cos \frac{4\pi}{3}$
 I) $\cot 300^\circ$ II) $\sec \frac{\pi}{3}$ III) $\csc \frac{-\pi}{4}$ IV) $\cos^{-1} \frac{-1}{2}$ V) $\sin^{-1} \frac{-\sqrt{3}}{2}$ VI) $\arctan 1$

- 20) Solve each of the following equations for θ , where $0 \leq \theta < 2\pi$.

i) $\sqrt{2} \cos \theta - 1 = -2$ ii) $2 \sin \theta - 1 = 0$

- 21) Use the graph of function f to evaluate each of the following.

i) $f(0) =$ iv) $f(4) =$
 ii) $f(-3) =$ v) $f(-2) =$
 iii) $f(3) =$ vi) $f(x) = 2$ when $x = ?$

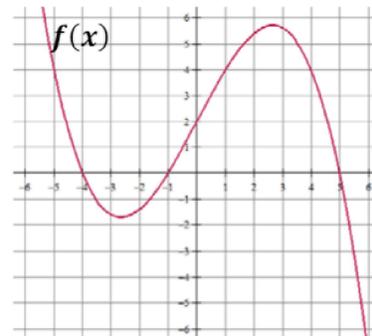


22)

Which of the following statements about the function $f(x)$ is true?

- I. $f(2) = 0$
 II. $(x + 4)$ is a factor of $f(x)$
 III. $f(5) = f(-1)$

- (A) I only
 (B) II only
 (C) III only
 (D) I and III only
 (E) II and III only



(23)

Consider the function: $f(x) = \frac{x^2 - 5x + 6}{x^2 - 4}$. Which of the following statements is true?

- I. $f(x)$ has a vertical asymptote of $x = 2$
- II. $f(x)$ has a vertical asymptote of $x = -2$
- III. $f(x)$ has a horizontal asymptote of $y = 1$

- (A) I only
- (B) II only
- (C) I and III only
- (D) II and III only
- (E) I, II and III

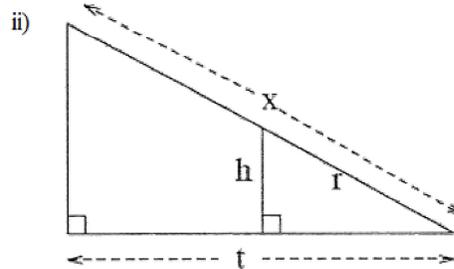
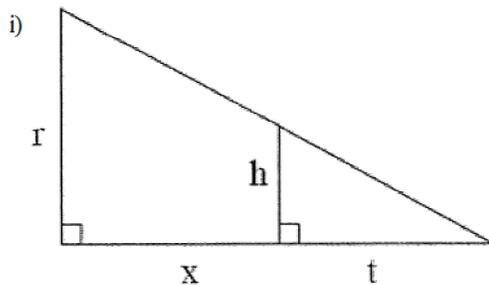
(24)

Use the properties of exponents/radicals to rewrite each of the following in the form of Ax^By^C , where A , B , and C are numbers.

i) $\frac{(2x^2)^3}{y}$ ii) $\sqrt{9xy^3}$ iii) $\frac{x(2/y)}{3/x}$ iv) $\frac{x^{-1}}{\sqrt[3]{x(y^{-2})}}$ v) $\frac{y^2\sqrt{25x^3}}{\sqrt[5]{y^2}}$

(25)

Solve for x in terms of the other variables for each of the following.



(26)

Make a rough sketch of the graph for each of the following. You must label at least 4 points on each.

- a. $f(x) = -2x + 5$
- b. $g(x) = \frac{1}{2}x^2$
- c. $h(x) = 5x^3$
- d. $f(x) = \sqrt{x}$
- e. $g(x) = \frac{1}{x}$
- f. $y = e^x$
- g. $g(x) = \sin x$
- h. $h(x) = \cos x$

AP Calculus AB
Answer Section

Summer Prep Packet

SHORT ANSWER

1 i. $[-2, 3)$ ii. $-4 < x \leq 5$ iii. $(-\infty, -2]$ iv. $(-\infty, \infty)$

2

i) $y - 5 = 4(x - 0)$ ii) $y - 71 = 5(x + 6)$ iii) $y - 16 = \frac{5}{7}(x - 4)$
iv) $y - 2 = -\frac{3}{2}(x + 1)$ v) $y = 3$ OR $y - 11 = \frac{5}{7}(x + 3)$

3

i) $(2, -1)$ ii) $(3, 3); (-2, 8)$

4

Answers may vary but would be equivalent to those below.

a. $P = \frac{A}{1+nr}$ b. $x = \frac{\pi}{\pi-1}$ c. $\frac{dy}{dx} = \frac{y-2x}{-2y-x}$ d. $y' = \frac{2x}{3y^2+2y-5}$

5 i) 6 ii) $x^2 + 12x + 38$ iii) $-2x - 2h + 1$ iv) $2x + h + 2$ v) $-2x^2 - 4x - 5$ vi) 18 vii) $g^{-1}(x) = \frac{x-1}{-2}$

6

i) $4x + 2\Delta x - 1$ ii) 3

7

a. $x \neq 7, -3$ b. $x \geq \frac{1}{2}$ c. $x > -3$

8

a. $f(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$ b. $g(x) = \begin{cases} x - 4, & \text{if } x \geq 4 \\ -(x - 4), & \text{if } x < 4 \end{cases}$ c. $h(x) = \begin{cases} 1, & \text{if } x > 0 \\ -1, & \text{if } x < 0 \end{cases}$

9

i) 4 ii) -3 iii) $\frac{1}{3}$ iv) -3

10

i) 13 ii) 9 iii) h

11

a. $(2x - 1)(x - 3)$ c. $x^4(x - 4)(x + 4)$
b. $(x - 1)(x + 1)(x^2 + 1)$ d. $(2x - 5)(2x + 5)(x - 2)$

12

a. $\frac{-3 \pm \sqrt{6}}{2}$ b. $x = \frac{1}{2}, -3$ c. $x = -\frac{1}{2}$

13 i) $(-\infty, -1]$ ii) $(2, \infty)$ iii) $(3, \infty)$ iv) $(-\infty, -4) \cup [2, \infty)$

14 i) $\frac{-7}{4}$ ii) -2 iii) -2 iv) 1

15 i) V.A. at $x = 4$ Hole at $x = 3$ ii) V.A. at $x = 0$ and $x = 1$ Hole at $x = -2$

16 $\frac{9\pi}{4} m^2$

17 $100\sqrt{5} km$

18 i) 30° or $\frac{\pi}{6}$ radians ii) 45° or $\frac{\pi}{4}$ radians

- (19) i) $\frac{-\sqrt{3}}{2}$ ii) $\frac{-\sqrt{2}}{2}$ iii) $\sqrt{3}$ iv) $\frac{-\sqrt{3}}{2}$ v) 1 vi) $\frac{-1}{2}$
 I) $\frac{-\sqrt{3}}{3}$ II) 2 III) $-\sqrt{2}$ IV) $\frac{2\pi}{3}$ V) $\frac{-\pi}{3}$ VI) $\frac{\pi}{4}$
- (20) i) $\frac{3\pi}{4}$ or $\frac{5\pi}{4}$ ii) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$
- (21) i) -4 ii) 0 iii) 2 iv) Undefined v) -2 vi) $x = -5$ and $x = 3$
- (22) E
- (23) D
- (24) i) $8x^6y^{-1}$ ii) $3x^{\frac{1}{2}}y^{\frac{3}{2}}$ iii) $\frac{2}{3}x^2y^{-1}$ iv) $1x^{\frac{-4}{3}}y^2$ v) $5x^{\frac{3}{2}}y^{\frac{8}{5}}$
- (25) Answers may vary but would be equivalent to the following.
- i) $x = \frac{rt}{h} - t$ ii) $x = \frac{tr}{\sqrt{r^2 - h^2}}$
- (26) Check your graphs in a graphing calculator or online at website such as Desmos. However, be sure you can graph these without any outside aids.